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TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 900

THE EFFECT OF THE MASSES OF THE CONTROLS ON THE LONGITUDINAL STABILITY WITH FREE ELEVATOR

By Rudolf Schmidt

Luftfahrtforschung
Vol. 16, No. 1, January 10, 1939
Verlag von R. Oldenbourg, München und Berlin

Washington
July 1939

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PART I

By Rudolf Schmidt

The longitudinal stability of an airplane is generally measured on model tests only for the condition of fixed elevator. The much more important stability in flight with free elevator is subsequently computed by substituting known values for the automatic adjustment of the free elevator under the effect of the air forces. In addition to these aerodynamic effects, the airplane in flight is also affected by the weight moments of all the control members on the elevator. The change in stability may be theoretically computed if the curve of these weight moments of the controls is known. In the present report, aerodynamic relations under the effect of the weight moments are investigated, and an example given of the computation of the stability for a practical case. Later, in Part II of this work, the effects of the masses of the controls on the dynamic longitudinal stability will be considered.

I. INTRODUCTION

It has long been known that the longitudinal stability of an airplane in flight with free elevator is influenced by the effect of the weight moments of the elevator control and the elevator. As far back as 1930, publications of the DVL appeared, in which were presented results of flight measurements showing the effect of various control conditions on the static longitudinal stability. With the aid of relatively simple computations, these effects may be explained and predicted from mechanical considerations. The effects acquire special significance when, for example, changes in the tail surface of an airplane are contemplated which would give rise to a change in the mass balance of the elevator. Such changes are often applied as a result of swinging tests. It is therefore desirable that the effect of such a change on the longitudinal stability be computed in advance.

*"Der Einfluss der Steuerungsmassen auf die Längsstabilität mit losem Ruder. Luftfahrtforschung, vol. 16, no. 1, January 10, 1939, pp. 31-37.

Another point of view is to be found from the fact that in a comparison between the stability as measured in a flight test and that computed on the basis of tests on the model, large differences arise, a considerable portion of which may be due to the unaccounted-for effect of the mass of the control parts. This fact should be particularly noted in the case of large airplanes, for on increasing the size of the aircraft the ratio of inertia forces of the individual control parts - which determine the automatic adjustment of the elevator and hence the stability - to the air forces, becomes larger since the latter forces must be practically independent of the size of the airplane in order that elevator control by human force may be at all possible. In order to make clear how strongly this effect enters as a phenomenon, a simple dimensional analysis will be considered.

If λ is the length ratio between two geometrically similar airplanes, the masses of the elevators are about in the ratio of λ^3. The masses of the controls increase to a lower power. For a push-rod control the ratio λ^2 will very nearly approach the true conditions since the push rods for the larger airplane are not only longer but also have larger diameters, in order to insure the required strength against buckling. Also the control columns in the case of large airplanes are generally arranged in pairs. For the entire control system including elevator, we may therefore consider a factor of increase $\lambda^{2.5}$.* Since it is a question, not of the control masses themselves but of the static moments exerted by them about the elevator hinge axis, the factor to be considered must be $\lambda^{3.5}$. On the basis of views held in Germany as to the admissible elevator control forces, it may be assumed that these may increase in the ratio $\lambda^{0.5}$ at any rate as a limit which, with the present-day airplanes has not yet been reached. The ratio of the weight moments to the air-force moments, therefore, increases as $\lambda^{3.5}/\lambda^{0.5} = \lambda^3$; i.e., the effect of the control masses increases approximately linearly with the weight in flight.

Although in the above analysis the magnification factor for the control forces was only roughly approximated,

*This value agrees very well with actual conditions, as is shown by a plot of the weights of the elevator control parts against the weight in flight of a large number of airplanes built by Dornier.

nevertheless it may be seen that even with a lower exponent, there is a considerable increase in the effect of the controls - which effect must be taken into account in the computation, particularly where there may be a lowering in the longitudinal stability.

In the present report the aerodynamic relations are investigated and a method given for computing the effects of the control masses. Another object of this paper is to provide a basis for the design of the controls, so as to avoid, as far as possible, a lowering of the longitudinal stability by unfavorable arrangement of the control parts.

II. THE LONGITUDINAL STABILITY WITH FREE ELEVATOR

Flight with free elevator is defined by the condition when no forces are exerted by the pilot on the elevator. This, however, does not mean that the air-force moment of the elevator is equal to zero, since in a practical case, even with free elevator, the latter is acted upon more or less by large forces which are due to the practically unbalanced masses of the individual control parts, as stick, control column, push rods, and levers, as well as the elevator itself. In the case of steady flight the effective air-force moment at the elevator must thus balance the resulting moment of all these mass effects. If, therefore, in any flight condition the air force and weight moments change for any reason whatever - for example, by a change in the flight-path inclination, when the airplane passes from level to climbing flight - then the elevator changes its position. This change in moment on the elevator naturally exerts exactly the same effect on the motion as a corresponding operation of the controls by the pilot. If these changes in moment for any change in flight condition are of such nature that the elevator of itself acts against the direction of the change in flight condition, then the effect of the moment acting on the elevator may be considered as automatic stabilization. Conversely, these moments exert a destabilizing effect if the elevator motion is such as to assist the change in the flight condition.

If it is assumed that all parts of the controls, including the elevator, are fully balanced by counterweights, so that no weight moments of any kind act on the elevator, then the elevator automatically adjusts itself under the effect of the air forces, and its air-force moment is equal to zero. Also, in this case, with changes in the flight condition the elevator changes its position and acts, according to the indication of its position, with stabilizing or destabilizing effect. Since this change in position depends only on the direction of flow, and not on the dynamic pressure, the angle of attack will, in what follows, be denoted as the determining factor for this purely aerodynamic effect.

Where weight moments of the controls are present, the effect may, in similar manner, be reduced to aerodynamic magnitudes which may be considered as the determining factors for the stabilizing or destabilizing effect of the elevator motion. If it is supposed that a part of these moments is independent of the position in space of the airplane, then - as will be shown in the following sections - the dynamic pressure will be the determining factor, but for moments that do depend on the position of the airplane, the pitch inclination will be the determining factor.

The three above-mentioned aerodynamic magnitudes: angle of attack, dynamic pressure, and pitch inclination are therefore sufficient to describe the motion of the free elevator and hence, to determine its effect on the longitudinal stability of the airplane.

In the following sections the effects of the weight moments of the controls - both those independent of the position of the airplane and those which change their magnitude with the pitch inclination of the airplane - will be investigated. For a more complete view of the entire problem of longitudinal stability with free elevator, it is desirable also to treat the case in which there is no effect of the weight moments - that is, where the angle of attack is the determining factor in the stability.

III. NOTATION

c_a, lift coefficient of the entire airplane.

c_w, drag coefficient of the entire airplane.

c_m, moment coefficient of the entire airplane (about the y axis).

c_R, moment coefficient of the elevator.

c_{nH}, normal force coefficient of the elevator.

α, angle of attack of the airplane.

α_H, angle of attack of the horizontal tail surface.

ϑ, angle of pitch.

γ, angle of path inclination.

β_H, elevator deflection.

v, velocity along flight path.

w, sinking velocity.

q_o, dynamic pressure along flight path.

q_H, dynamic pressure at horizontal tail surface.

F, wing area.

F_H, horizontal tail surface area.

F_R, elevator area.

t, reference chord of wing.

t_R, elevator chord.

l_H, distance from force at tail surface to center of gravity.

G, gross weight.

M_R, elevator moment.

c_s, propeller thrust coefficient referred to the propeller disk area F_s.

Other notations explained in the text.

IV. THE DYNAMIC PRESSURE AS DETERMINING FACTOR

There is first assumed the simplest case in which the moments acting on the elevator are only such as are independent of the angle of attack and position of the airplane in space. A control of this kind may also be realized in practice. It is sufficient, by a suitable choice of elevator shape - the properties of which will not here be gone into further - to take care that the elevator does not move with any changes in the angle of attack, so that it behaves like a fixed elevator. Furthermore, all parts of the controls are so balanced by counterweights that no moments act on the elevator. If now there is introduced a force whose moment, referred to the elevator, is always constant, then we obtain a control of the type desired. This type of control is schematically represented in figure 1. The additional applied force we shall assume as produced by the weight G' which, with changes in the pitch, always adjusts itself in the direction of gravity, and by means of a rope and pulley, exerts a turning moment $M_R = G' h$, also independent of the elevator setting.

If the airplane is first considered to be in an equilibrium state, without the effect of an additional moment, and if a moment of this kind is then applied, the elevator is deflected by an amount $\Delta\beta_H$, and thereby changes the pitching moment coefficient c_m of the airplane. This change in the value of c_m is of the amount

$$\Delta c_m = \frac{\Delta c_{n_H} F_H l_H q_H}{F t q_o} \tag{1}$$

The change in the normal force coefficient of the horizontal tail surface for an elevator deflection $\Delta\beta_H$ is

$$\Delta c_{n_H} = \frac{\partial c_{n_H}}{\partial \beta_H} \Delta\beta_H \tag{2}$$

Since the air-force moment at the elevator balances the additional applied moment M_R, there is obtained the elevator moment coefficient

$$c_R = \frac{M_R}{F_R t_R q_H} = \frac{\partial c_R}{\partial \beta_H} \Delta\beta_H \tag{3}$$

hence,

$$\Delta\beta_H = \frac{M_R}{F_R\ t_R\ q_H}\ \frac{\partial\beta_H}{\partial c_R} \tag{4}$$

Substituting in equation (2) there is obtained:

$$\Delta c_{n_H} = \frac{\partial c_{n_H}}{\partial\beta_H}\ \frac{M_R}{F_R\ t_R\ q_H}\ \frac{\partial\beta_H}{\partial c_R}$$

and therefore from equation (1):

$$\Delta c_m = \underbrace{\frac{F_H\ l_H}{F\ t\ F_R\ t_R}}_{A}\ \underbrace{\frac{\partial c_{n_H}}{\partial c_R}}_{B}\ \underbrace{\boxed{\frac{M_R}{q_o}}}_{C} \tag{5}$$

In the above equation, A represents only structural magnitudes, B is an aerodynamic coefficient of the tail surface and also depends only on the shape, and C is the variable factor of the additional moment. It may be seen that for a given additional moment M_R, the change in the value of c_m for the airplane depends only on the aerodynamic pressure q_o. We shall denote this as "dynamic pressure stabilizing." If, in equation (5) $1/F\ q_o$ is replaced by c_a/G, then the equation becomes:

$$\Delta c_m = \frac{F_H\ l_H}{t\ F_R\ t_R\ G}\ \frac{\partial c_{n_H}}{\partial c_R}\ M_R\ \boxed{c_a} \tag{6}$$

The change in stability is then obtained by differentiation as

$$\frac{d(\Delta c_m)}{d\ c_a} = \frac{F_H\ l_H}{t\ F_R\ t_R\ G}\ \frac{\partial c_{n_H}}{\partial c_R}\ M_R \tag{7}$$

It may thus be seen that the stability is changed independent of the flight condition (throttle setting, angle of attack). Plotting c_m against c_a (fig. 2), it is seen that under the effect of the additional moment M_R, the slope of the curve is changed by the amount given by equation (7). The curves intersect at point $c_a = 0$ since, according to equation (6) for $c_a = 0$, Δc_m also equals zero.

V. THE ANGLE OF ATTACK AS THE DETERMINING FACTOR

The assumption made in the previous section - that the elevator behaves as a fixed elevator - applies only to certain cases. In general, a normal flap elevator adjusts itself in a destabilizing sense, although for horn and other external balancing, a stabilizing automatic adjustment is also possible. With hinge axis shifted backward ("internal" balance), the condition may be attained where the automatic adjustment is not exactly equal to zero.

In what follows, we shall investigate the general case in which the angle of attack acts as the determining factor for the automatic adjustment of the elevator. For this purpose we shall again think of the control as schematized in the manner shown in figure 3.

The elevator which, as in the previous section, we shall assume has no automatic adjustment, is coupled by a suitable rod to an auxiliary surface F_z, freely situated in the air stream. (Whether this auxiliary surface is located ahead of or behind its hinge axis, is of no importance for our consideration as only the sign is affected.) This auxiliary surface exerts on the elevator an air-force moment of the amount

$$M_z = c_1 \; c_{n_z} F_z \, l_z \, q_H$$

where c_1 is a factor representing the lever transmission.

Proceeding similarly, as in the previous section, there is again obtained:

$$\Delta \beta_H = \frac{M_R}{F_R \; t_R \; q_H} \frac{\partial \beta_H}{\partial c_R} \tag{4}$$

Substituting for M_R the additional moment M_z of the auxiliary surface, there is obtained:

$$\Delta \beta_H = \frac{c_1 \; c_{n_z} \; F_z \; l_z \; q_H}{F_R \; t_R \; q_H} \frac{\partial \beta_H}{\partial c_R} \tag{5}$$

where c_{n_z} is a function of α_H and $\Delta\beta_H$.

Setting

$$c_{n_z} = \frac{\partial c_{n_z}}{\partial \alpha_H} \alpha_H + \frac{\partial c_{n_z}}{\partial \beta_H} \Delta\beta_H \qquad (6)$$

and for

$$\frac{\partial c_{n_z}}{\partial \beta_H} = \frac{1}{c_1} \frac{\partial c_{n_z}}{\partial \alpha_H}$$

and substituting in equation (5), there is obtained:

$$\Delta\beta_H = \frac{\frac{F_z}{F_R} \frac{l_z}{t_R} c_1 \frac{\partial \beta_H}{\partial c_R} \frac{\partial c_{n_z}}{\partial \alpha_H} \alpha_H}{1 - \frac{F_z}{F_R} \frac{l_z}{t_R} \frac{\partial \beta_H}{\partial c_R} \frac{\partial c_{n_z}}{\partial \alpha_H}} \qquad (7)$$

Setting $\frac{F_z}{F_R} \frac{l_z}{t_R} = A_1$ and substituting equation (7) in equations (2) and (1), we have:

$$\Delta c_m = \underbrace{\frac{F_H}{F} \frac{l_H}{t} \frac{\partial c_{n_H}}{\partial \beta_H} \left[\frac{A_1 \ c_1 \ \frac{\partial \beta_H}{\partial c_R} \frac{\partial c_{n_z}}{\partial \alpha_H}}{1 - A_1 \frac{\partial \beta_H}{\partial c_R} \frac{\partial c_{n_z}}{\partial \alpha_H}} \right]}_{A} \underbrace{\boxed{\frac{\alpha_H \ q_H}{q_o}}}_{B} \qquad (8)$$

where A again is a factor which depends only on the external shape, while B is the variable factor. It may be seen that the change in moment Δc_m depends, besides on the dynamic pressure ratio q_H/q_o, also on the angle of attack α_H of the horizontal tail surface; and since this is a function of the angle of attack α, it depends on the latter. We denote this as "angle of attack, stabilizing." In order to investigate how the stability dc_m/dc_a changes, we must transform the factor $B = \frac{\alpha_H \ q_H}{q_o}$ in such a manner that only c_a appears as the variable. We shall here investigate two cases:

a) The theoretical case where the auxiliary elevator surface is not located in the region of influence of the propeller slipstream and the wing downwash;

b) The more practical case where the above condition does not apply.

We shall therefore investigate how the propeller slipstream and downwash alter the conditions:

a) In the first case, $q_H = q_o$ and $\alpha_H = \alpha$. The factor B thus becomes $B = \alpha$. Replacing α by $\frac{\partial\alpha}{\partial c_a} c_a$, there is obtained by differentiation with respect to c_a:

$$\frac{d(\Delta c_m)}{dc_a} = A \frac{\partial\alpha}{\partial c_a}$$

i.e., the change in stability depends only on the structural magnitudes and on $\partial\alpha/\partial c_a$, which is determined by the shape (aspect ratio), but does not depend on the flight condition. An "angle of attack - stabilizing" of this kind therefore behaves exactly as the "dynamic pressure stabilizing."

b) In the second case, there must be substituted for α_H and q_H/q_o functions of c_a, which include the effects of the slipstream and downwash.

The dynamic-pressure ratio q_H/q_o is, according to the jet theory, equal to $1 + c_s$ where $c_s = S/F_s\ q_o$ is the thrust loading of the propeller. The thrust loading c_s may, with good approximation, be set proportional to c_a, particularly for small changes in c_s. We then have:

$$\frac{q_H}{q_o} = 1 + K_1\ c_a$$

The angle of attack α_H is similarly proportional to c_a:

$$\alpha_H = K_2\ c_a$$

and expression B in equation (8) then becomes:

$$B = \alpha_H \frac{q_H}{q_o}$$

$$= K_2 \ c_a + K_1 \ K_2 \ c_a{}^2 \qquad (9)$$

Putting this expression for B in equation (8) and differentiating with respect to c_a, there is obtained the change in stability:

$$\frac{d(\Delta c_m)}{dc_a} = A \ K_2 \ (1 + 2 \ K_1 \ c_a) \qquad (10)$$

It may be seen that in this case the change in stability is no longer independent of the flight condition. The change in stability becomes larger, the larger is c_a. It changes with the throttle setting represented by the factors K_1 and K_2, where the former includes the effect of the throttle setting, and the latter, the total downwash effect.

Plotting c_m against c_a (fig. 4), the change may be split into two portions - one varying linearly with c_a, the other varying as $c_a{}^2$. It may be seen that with "angle of attack stabilizing" there is an additional term depending on $c_a{}^2$. This has the result that the "angle of attack stabilizing," particularly at high values of c_a, becomes more energetically effective than the "dynamic pressure stabilizing." This kind is generally the one that occurs in practice. The relation $c_{n_z} = \frac{\partial c_{n_z}}{\partial \alpha_H} \alpha_H + \frac{\partial c_{n_z}}{\partial \beta_H} \beta_H$ assumed for the effect of the auxiliary surface for the scheme represented, may be applied to every elevator regardless of the shape of elevator or of how its force balance is obtained. The relations obtained therefore are generally valid and not conditioned on the existence of an auxiliary surface of this kind, which serves only for a better explanation of the process. The theory may be confirmed repeatedly in practice. If the value c_m is measured in a flight test as a function of c_a, there will almost always be obtained a deviation from the linear law - which deviatiom is explained by the above theoretical investigation.

VI. THE PITCH INCLINATION AS DETERMINING FACTOR

A type of control in which additional forces or moments are applied that depend on the position of the airplane in space, is schematically shown in figure 5. A gravity pendulum is installed in the airplane which, during the change in pitch ϑ, introduces a weight moment M_z into the control. This moment is $M_z = c_1 \; G_z \; l_z \; \sin \epsilon$ (c_1 is the transmission factor). Proceeding in the same manner as in sections IV and V, we again have:

$$\Delta \beta_H = \frac{M_R}{F_R \; t_R \; q_H} \frac{\partial \beta_H}{\partial c_R} \tag{4}$$

Substituting

$$M_R = c_1 \; G_z \; l_z \; \sin \left(\vartheta - \frac{1}{c_1} \; \Delta \beta_H \right)$$

which, for small angles ϑ and $\Delta\beta_H$, may approximately be written:

$$M_R = c_1 \; G_z \; l_z \; \vartheta - G_z \; l_z \; \Delta \beta_H \tag{5}$$

In equation (4), we have:

$$\Delta \beta_H = \frac{c_1 \; G_z \; l_z \; \vartheta}{F_R \; t_R \; q_H} \frac{\partial \beta_H}{\partial c_R} - \frac{G_z \; l_z \; \Delta \beta_H}{F_R \; t_R \; q_H} \frac{\partial \beta_H}{\partial c_R}$$

and putting $\frac{G_z \; l_z}{F_R \; t_R} = A_2$, there is obtained:

$$\Delta \beta_H = \frac{c_1 \; A_2 \; \frac{\partial \beta_H}{\partial c_R} \; \vartheta}{q_H + A_2 \; \frac{\partial \beta_H}{\partial c_R}} \tag{6}$$

which, substituted in equations (1) and (2), gives:

$$\Delta c_m = \underbrace{\frac{F_H \; l_H}{F \; t} \frac{\partial c_{n_H}}{\partial c_R} \; c_1 \; A_2}_{A} \; \underbrace{\boxed{\frac{\vartheta \; \frac{q_H}{q_o}}{q_H + A_2 \; \frac{\partial \beta_H}{\partial c_R}}}}_{B} \tag{7}$$

The magnitude A again depends on the external shape, while B is the variable factor including, in addition to the dynamic pressures q_o and q_H, the pitch ϑ. We therefore designate this stabilizing as the "pitch inclination stabilizing."

To investigate the magnitude of the change in the stability $d(\Delta c_m)/dc_a$, we must again transform the term B in equation (7) into an expression in which only c_a appears.

Here, too, as in the case of "angle of attack stabilizing," we shall investigate the two cases, namely: one in which the tail surface is located in a region with no propeller slipstream effect; and the other more practical case, where there is such an effect.

a) In the first case we again have $q_H = q_o$, and then $B = \dfrac{\vartheta}{q_o + A_2 \dfrac{\partial \beta_H}{\partial c_R}}$. The pitch ϑ is then made up of the angle of attack and the flight-path angle γ. We shall consider only the case where $\gamma > 0$.

In climb:

$$\gamma = \frac{w}{v} = \frac{c_s - c_w}{c_a}$$

The thrust coefficient c_s (here referred to the wing area) is, to a first approximation, proportional to c_a. The drag coefficient is made up of the induced portion and the remaining drag $c_w{'}$. We then have:

$$\gamma = \frac{K_1\ c_a - (K_2\ c_a^2 + c_w{'})}{c_a}$$

The pitch ϑ is obtained from $\vartheta = \gamma + \alpha$.

Setting $\alpha = K_3\ c_a$, we have:

$$\vartheta = K\ c_a - \frac{c_w{'}}{c_a} + K_1 \quad (K_3 - K_2 = K)$$

Putting $q_o = \dfrac{G}{F}\dfrac{1}{c_a}$, equation (7) becomes:

$$\Delta c_m = A \left(\frac{K\ c_a^2 - c_w' + K_1\ c_a}{\frac{G}{F} + A_2 \frac{\partial \beta_H}{\partial c_R}\ c_a} \right) \tag{8a}$$

By differentiation, there is obtained an equation of the form

$$\frac{d(\Delta c_m)}{d\ c_a} = \frac{a_1\ c_a^2 + b_1\ c_a + c_1}{d_1\ c_a^2 + e_1\ c_a + f_1} \tag{9a}$$

In the above a_1, b_1, c_1, d_1, e_1, f_1 are coefficients, in which are contained the structural magnitudes and the aerodynamic coefficients depending on them.

The derivation of the equations for level flight $\gamma = 0$ and climbing flight $\gamma < 0$, offers nothing essentially new. The expressions for

$$\frac{d\ (\Delta c_m)}{d\ c_a}$$

merely contain other coefficients.

For level flight, there is obtained:

$$\frac{d(\Delta c_m)}{d\ c_a} = \frac{a_2\ c_a^2 + b_2\ c_a}{d_2\ c_a^2 + e_2\ c_a + f_2} \tag{9b}$$

and for gliding flight:

$$\frac{d(\Delta c_m)}{d\ c_a} = \frac{a_3\ c_a^2 + b_3\ c_a + c_3}{d_3\ c_a^2 + e_3\ c_a + f_3} \tag{9c}$$

Equations (9a,b,c) show that for all three flight conditions considered, the change in stability is also determined by the structural magnitudes. This becomes clear when account is taken of the fact that the pitch inclination ϑ is not a unique function of c_a as are the dynamic pressure and angle of attack. For equal c_a, the pitch may be positive or negative, depending on whether the airplane climbs or descends. The changes in stability can be specified, therefore, only for individual cases depending on the external relations.

b) The derivation of the equations for the second case, in which the tail is situated in the region of influence of the propeller slipstream, gives no essential change. In the equation there merely occurs an additional term with c_a^3 in the numerator. The equation for climb is:

$$\frac{d(\Delta c_m)}{d\ c_a} = \frac{a\ c_a^3 + b\ c_a^2 + c\ c_a + d}{e\ c_a^2 + f\ c_a + g} \tag{9d}$$

VII. CARRYING OUT OF THE COMPUTATION

In the computation of the effects of the control weights, we shall assume the automatic adjustment of the elevator under the effect of the angle of attack as known. This "angle of attack stabilizing" may also always be estimated from a model test with fixed elevator, taking into consideration the values obtained for the elevator shapes from the experimental and theoretical results.

A simple computational treatment of "dynamic pressure stabilizing" is possible by means of the equations derived in section IV. The aerodynamic coefficient $\frac{\partial c_{n_H}}{\partial c_R}$ may be obtained from the Glauert theory or from model tests. It is better, naturally, for this value to be determined directly from flight test, as is possible without difficulty.

A computational treatment of "pitch inclination stabilizing" in a general form would involve, however, a large expenditure of time. The computation may be considerably simplified if the moment on the elevator is determined as a function of the pitch and the elevator deflection. This can easily be done for a given type of control since the weights of the individual control parts and their positions of gravity may be estimated with sufficient accuracy.

The computation procedure is as follows: We compute from the c_m/c_a curve, which is determined for a free elevator from a model test or by computation, the change in c_m if the resultant moment of all control parts at the elevator axis is a known function of the pitch inclination and elevator deflection. From this function we compute for a definite flight condition (c_a, γ) the effective weight moment first under the assumption that β_H does not change. This moment we substitute in equation (4) and obtain the change in elevator position $\Delta\beta_H$. From the function $M_R = f(\beta_H, \gamma)$, the actual effective moment M_R may be determined. The latter we substitute in equation (5) and thus obtain for each flight condition a point of the new c_m/c_a curve. By graphical differentiation, there is obtained therefrom the change in stability.

VIII. EXAMPLE

We shall carry out this computation for a practical case, namely, for a full throttle condition in the region of best climb up to $c_{a_{max}}$, and also for the idling condition up to vertical diving.

The airplane data are the following:

$G = 6,500$ kg

$F = 55$ m^2

$F_H = 9.5$ m^2

$F_R = 3.1$ m^2

$t_R = 0.7$ m

$l_H = 9.5$ m

$F_S = 2 \times 8.05$ m^2

$N_V = 2 \times 625$ hp. at full throttle.

The tail coefficients $\frac{\partial c_{n_H}}{\partial c_R} = 7.42$ and $\frac{\partial \beta_H}{\partial c_R} = 3.80$ are to be obtained from a model test of the tail surface. They may also be approximately computed from similar model tests or from the Glauert theory.

The dependence of the resultant weight moment of the elevator control on the pitch and the elevator deflection, is shown in figure 6 both for the control with fully balanced elevator and for over- and underbalance of the elevator by ±250 kg cm. Dependence on the elevator deflection is mainly to be explained by the effect of the control column, since the weights of the thrust rods running along the fuselage only give a dependence on the pitch.

The pitch angles corresponding to the individual flight conditions are determined by the familiar flight methods. In a similar manner there is obtained the angle of attack so that the pitch inclination may be computed.

The example is computed for three cases:

1. The elevator is fully balanced; i.e., its center of gravity coincides with the hinge axis. The control effects are due only to the control column and control connections.

2. By means of a weight at the elevator, a moment of -250 kg cm is applied in the "push" direction, the center of gravity thus lying behind the hinge axis.

3. By means of a weight at the elevator, a moment in the "pull" direction of +250 kg cm is applied, the center of gravity of the elevator thus lying ahead of the hinge axis.

In figures 7 and 8, the c_m/c_a curves are shown for the two flight conditions. By graphic differentiation the stability derivative dc_m/dc_a is determined and plotted in figures 9 and 10. The example shows that although the effect of the controls without elevator is so large that for an airplane whose stability as a result of large propeller slipstream and downwash effects is relatively small - as in the case of almost all modern high-speed airplanes - the stability at large values of c_a may become negative for full-throttle climbing flight. By applying a weight moment in the "push" direction, the stability may be increased and the effect partially balanced out. It should be noted that an additional weight moment of 250 kg cm at the elevator for a 6-ton airplane is in no way disturbing to the flight.

A weight moment in the "pull" direction always acts to decrease the stability. This is of particular importance when, for reasons of safety against vibration, it is necessary to bring about a mass balance at the elevator. This decrease in stability, naturally, may again be balanced by a suitable counterweight in the controls.

As follows from the derived formulas, the effect of the weight moments is a linear function of the aerodynamic coefficient $\frac{\partial c_{n_H}}{\partial c_R}$. This, however, means that for a lowering in c_R - that is, for a lowering in the elevator forces, there is an increase in the effects described. The instability may, for example, be increased by increasing the force balance for an elevator whose center of gravity lies behind the hinge axis, and conversely.

IX. CONCLUDING REMARKS

As a result of the strong increase in the wing loading and hence in the downwash and slipstream effects which reduce the longitudinal stability, modern high-speed airplanes - particularly, for full-throttle conditions - possess in general only a slight reserve of longitudinal stability. On the other hand, present requirements as to blind flying demand a certain measure of stability. The practical computed example, confirmed by measurements, shows what an important and deciding effect the weight moments of the control parts have on the stability. Without taking this into account, every prediction of the stability from model tests is questionable. The designer, too, must from the beginning take care that by proper kinematic arrangement of the control parts, the effect of the control weights is to raise the stability of the airplane. The weight balance of the control parts that may subsequently be required, always leads to an undesirable increase in weight, which may most frequently be ascribed to ignorance of the relations described.

Translation by S. Reiss,
National Advisory Committee
for Aeronautics.

(a) Elevator weight moment = -250 kgcm (under balance).

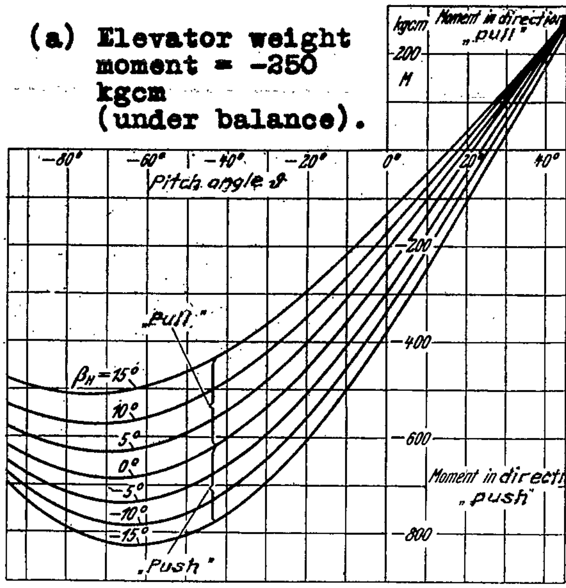


Figure 6

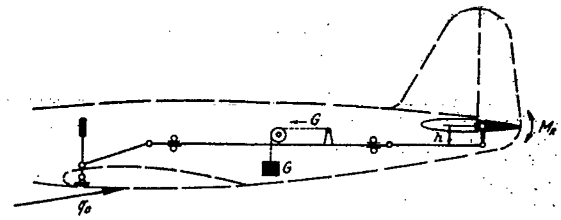


Figure 1.- Sketch of "dynamic pressure stabilizing."

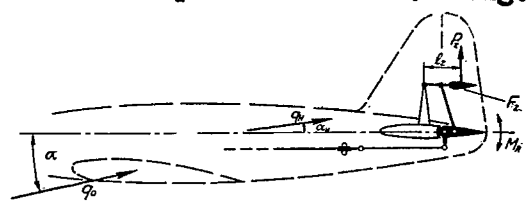


Figure 3.- Sketch of "angle of attack stabilizing."

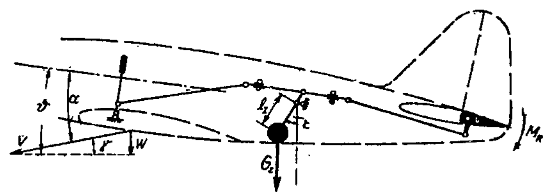
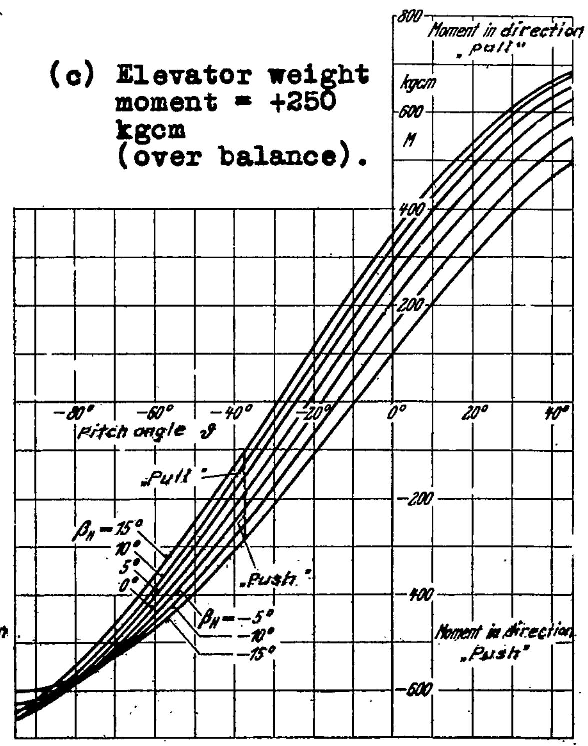


Figure 5.- Sketch of "pitch inclination stabilizing."

(c) Elevator weight moment = +250 kgcm (over balance).



(b) Elevator weight moment = 0 kgcm.

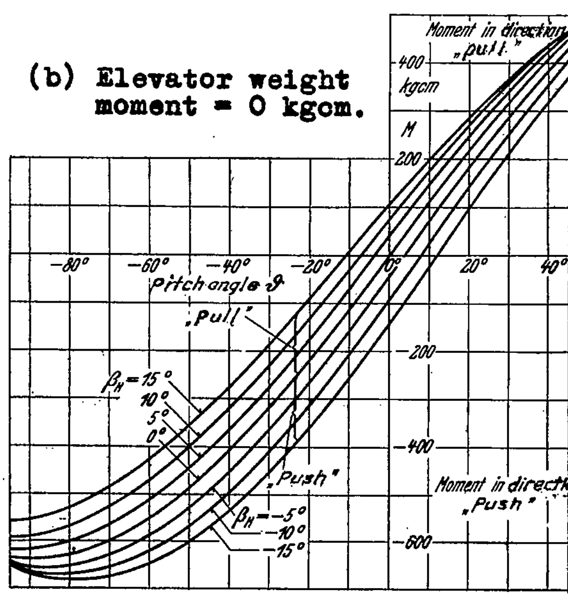


Figure 6.- Weight moments for vertical control.

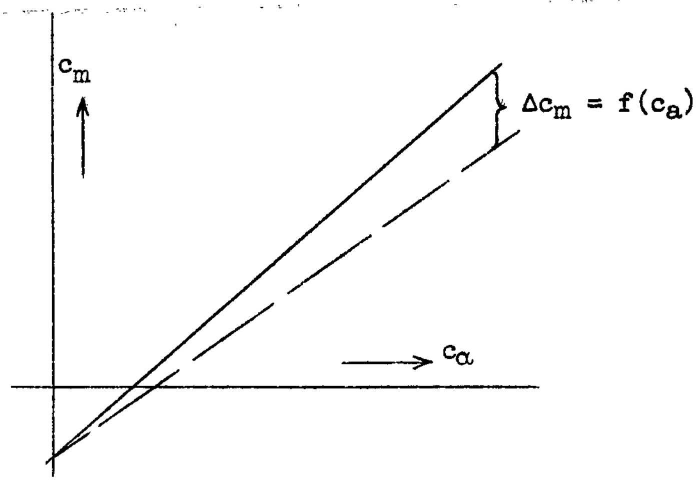


Figure 2.- "Dynamic pressure stabilizing."

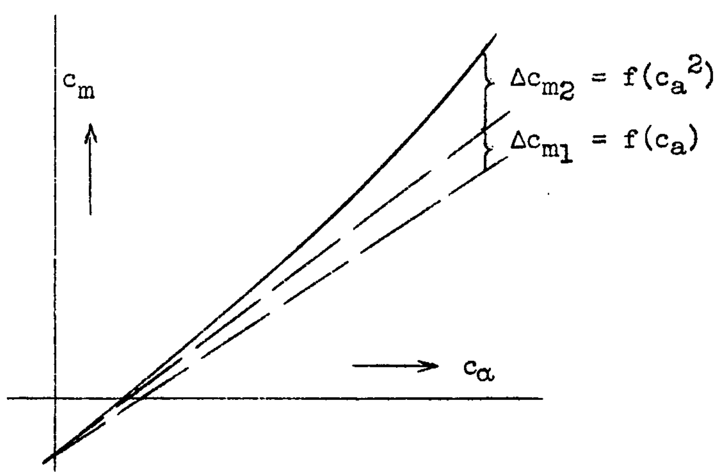


Figure 4.- "Angle of attack stabilizing."

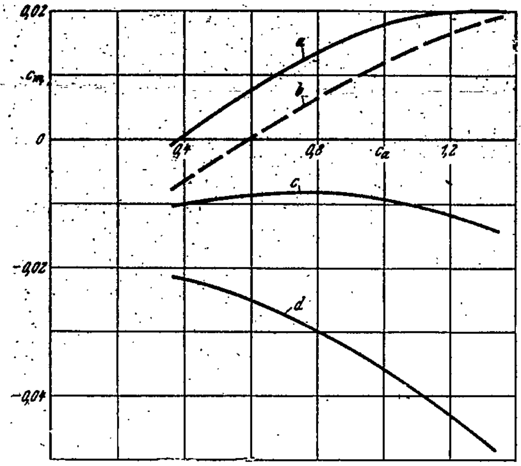


Figure 7.- Curves of c_m against c_a, full throttle.

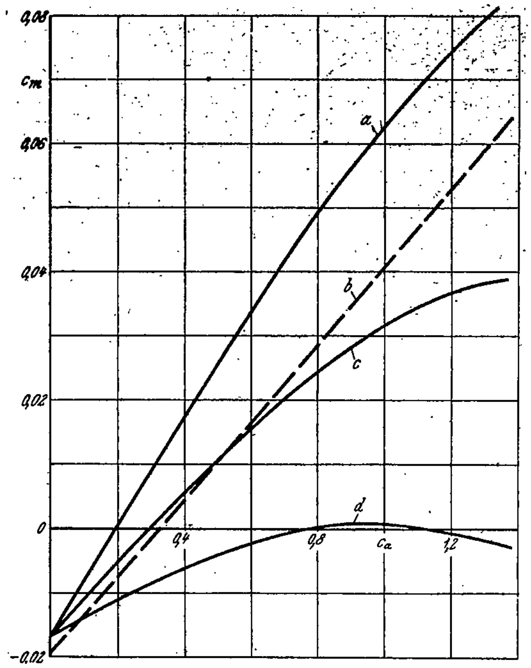


Figure 8.- Curves of c_m against c_a, idling.

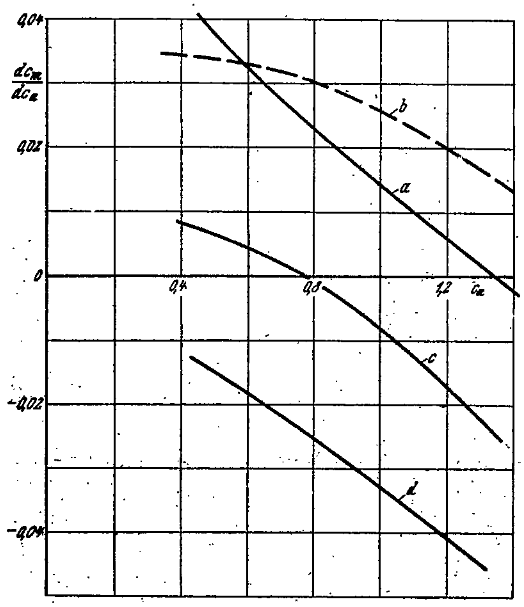


Figure 9.- Longitudinal stability dc_m/dc_a at full throttle.

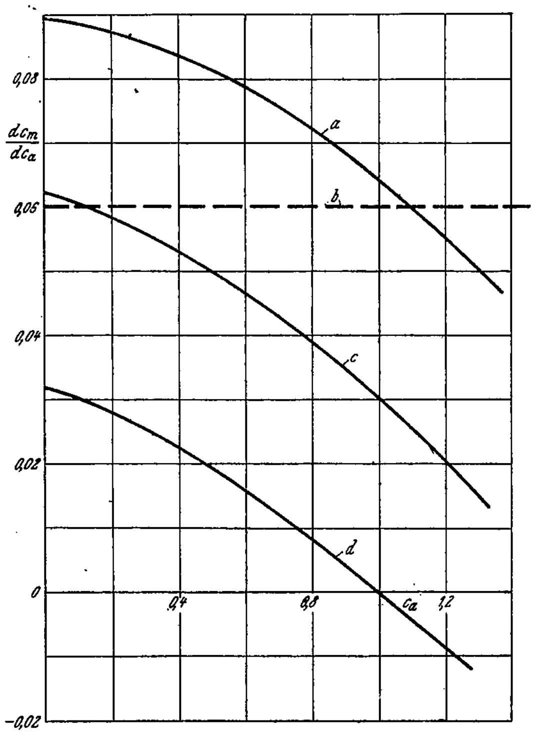


Figure 10.- Longitudinal stability dc_m/dc_a for idling.

In figs. 7-10:

(a) elevator -250 kgcm (under balance), controls not balanced.

(b) elevator and controls balanced.

(c) elevator balanced, controls not balanced.

(d) elevator +250 kgcm (over balance), controls not balanced.